AP Calculus AB — WS 91 - Motion & Rates Review

1) $v(t) = -1 + e^{1-t}$

a)
$a(t) = -e^{1-t}$
$a(3) = -e^{-2}$

b) $v(3) = -1 + e^{-2} = -1 + \frac{1}{e^2} < 0$

$a(3) = -\frac{1}{e^2} < 0$

Since $v(3)$ & $a(3)$ have the same sign, the speed of the particle is increasing.

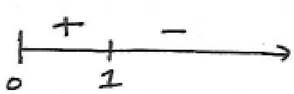
c) $-1 + e^{1-t} = 0$

$e^{1-t} = 1$

$1 - t = \ln 1$

$1 - t = 0$

$t = 1$



Since $v(t)$ Δs signs at $t=1$, the particle Δs directions

d)
Total Distance = MOVEMENT RIGHT + MOVEMENT LEFT

$$= \int_0^1 (-1 + e^{1-t})dt + \left| \int_1^3 (-1 + e^{1-t})dt \right|$$

$$= -t - e^{1-t} \Big|_0^1 + \left| (-t - e^{1-t}) \Big|_1^3 \right|$$

$$= (-1 - 1) - (-e^1) + \left| (-3 - e^{-2}) - (-1 - 1) \right|$$

$$= -2 + e + \left| \left(-3 - \frac{1}{e^2}\right) + 2 \right|$$

$$= -2 + e + \left| -1 - \frac{1}{e^2} \right|$$

2) a) $a(36) \approx \dfrac{v(40) - v(32)}{40 - 32}$

$$= \frac{7 - (-4)}{8} = \frac{11}{8} \ m/sec^2$$

b) $\int_{20}^{40} v(t)dt$ represents the displacement of the particle, in meters, from $t = 20$ sec to $t = 40$ sec.

$$\approx \frac{1}{2}\left[v(20) + v(25)\right]\cdot(5) + \frac{1}{2}\left[v(25) + v(32)\right]\cdot(7) + \frac{1}{2}\left[v(32) + v(40)\right]\cdot(8)$$

$$= \frac{5}{2}\left[-10 - 8\right] + \frac{7}{2}\left[-8 - 4\right] + 4\left[-4 + 7\right] = -75 \ m$$

c) Since $v(t)$ is differentiable, IVT applies.
If $v(t)$ Δs signs, then the particle changes direction

$\left.\begin{array}{l} v(8) = 5 > 0 \\ v(20) = -10 < 0 \end{array}\right\}$ $v(t)$ Δs signs $\quad$ $\left.\begin{array}{l} v(32) = -4 < 0 \\ v(40) = 7 > 0 \end{array}\right\}$ $v(t)$ Δs signs

Change of direction in $(8, 20)$ & $(32, 40)$

d) $a(t) > 0$ for $(0, 8)$ $\rightarrow$ velocity is increasing

If $v(t)$ is constant.

$V(t) = 3$ on $(0, 8)$

position would be:

$$x(8) = 7 + \int_0^8 3\,dt$$

$$= 7 + 24$$

$$= 31$$

But $v(t)$ is increasing:

So: $x(8) \geq 31$

which is greater than 30.

3) a) Enter Rate: 8 gal/min

Exit Rate: $\sqrt{t+1}$ gal/min

At $t=0$: 30 gal

$$\int_0^3 (t+1)^{1/2} dt$$

$$\frac{2}{3}(t+1)^{3/2}\Big|_0^3$$

$$\frac{2}{3}(8) - \frac{2}{3} = \frac{14}{3} \text{ gal}$$

b) $W(t) \rightarrow$ Amount of H_2O

$$W(3) = 30 - \int_0^3 (t+1)^{1/2} dt + \int_0^3 8\,dt$$

$$= 30 - \frac{14}{3} + 24$$

$$= \frac{148}{3} \text{ gal}$$

4) Enter: $H(t)$

Exit: $R(t)$

$t=0$: 125 gal

a) $\int_0^{12} H(t)\,dt = 70.571$ gal

b) $H(6) = 5.394$ gal/hr

$R(6) = 8.319$ gal/hr

Since $\underset{\text{(Enter Rate)}}{H(6)} < \underset{\text{(Exit Rate)}}{R(6)}$, the level of oil is falling

c) $125 + \int_0^{12} H(t)\,dt - \int_0^{12} R(t)\,dt = 122.026$